

The Skeleton Problem

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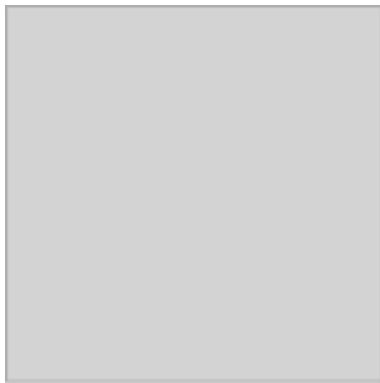
School of Mathematics and Statistics
The University of Melbourne

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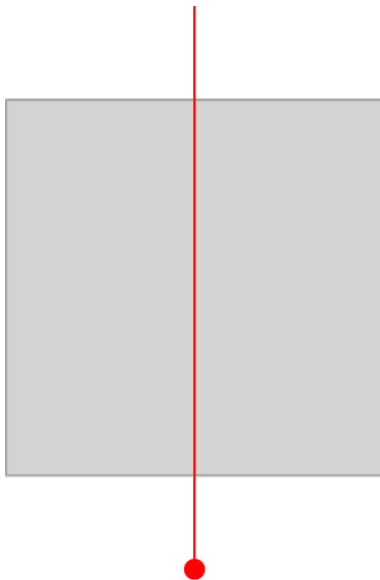
Constructing minimum Euclidean skeletons of polygonal regions with an unbounded number of holes is NP-hard.

Joint work with Charl Ras and Marcus Brazil.

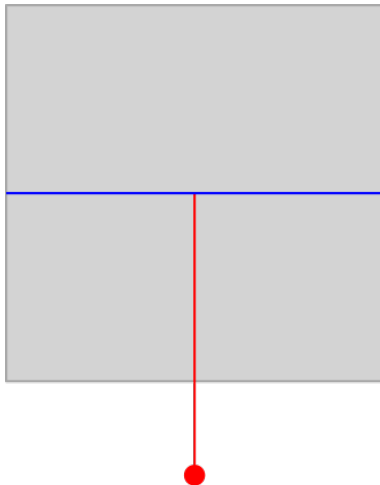
An Easy Example



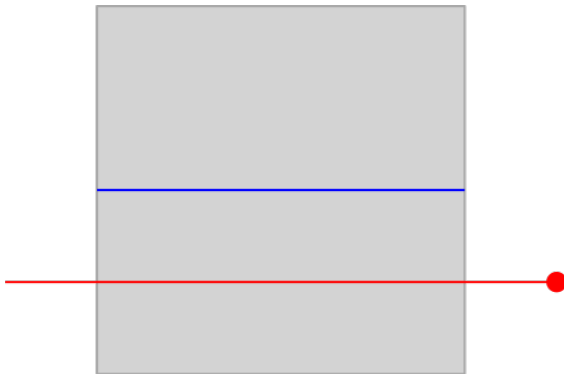
An Easy Example



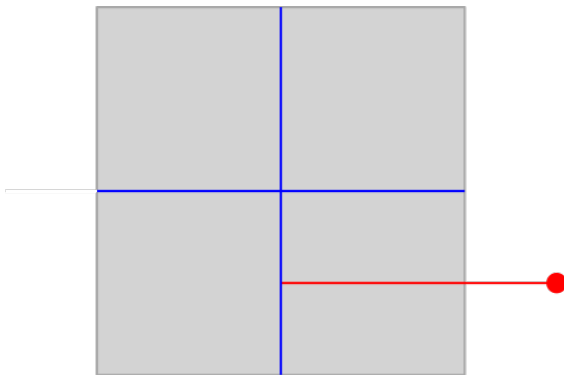
An Easy Example



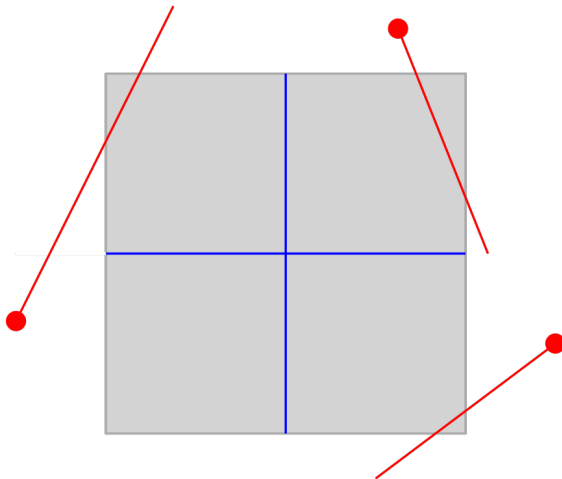
An Easy Example

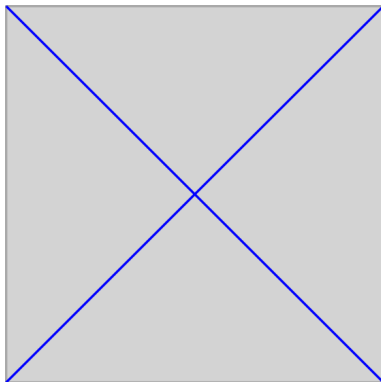


An Easy Example

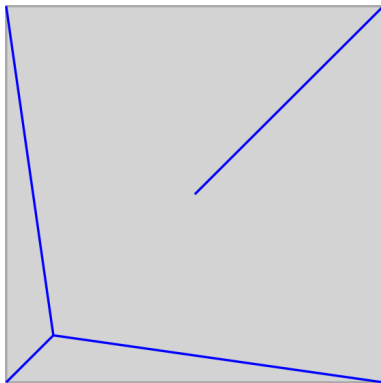


An Easy Example



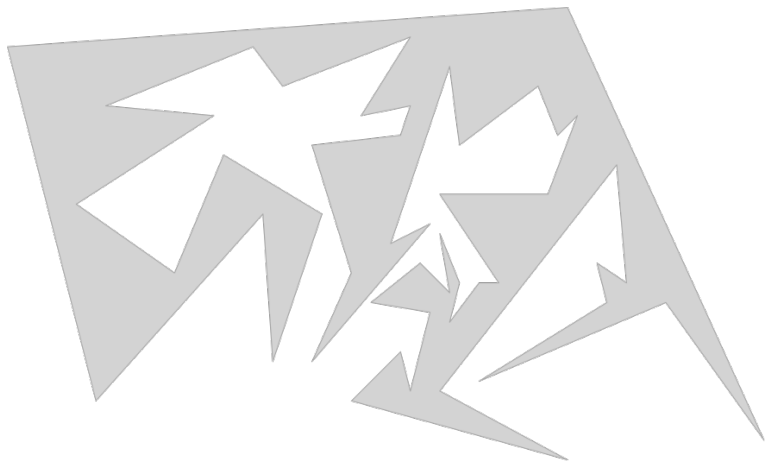


Solution to the Skeleton Problem for a square.



The conjectured solution to the Opaque Cover Problem for a square.

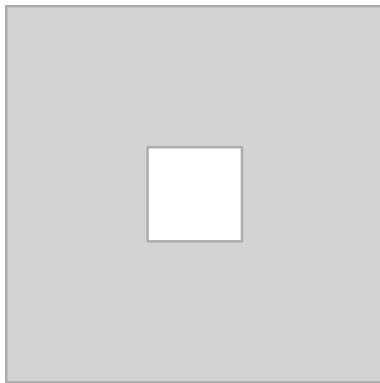
A More Complex Example



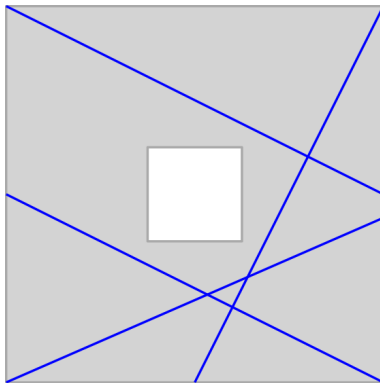
What is the minimum skeleton of this polygon?

- ▶ Intersects every convex vertex
- ▶ Connected

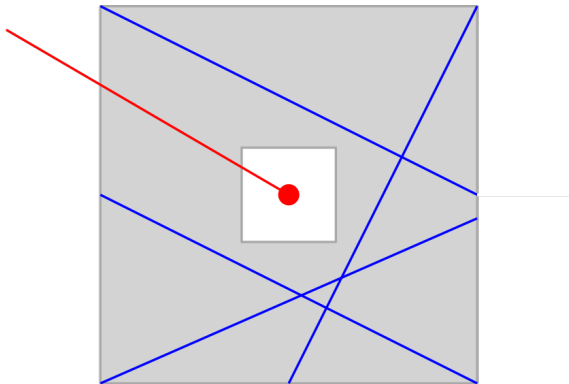
Example with a Hole

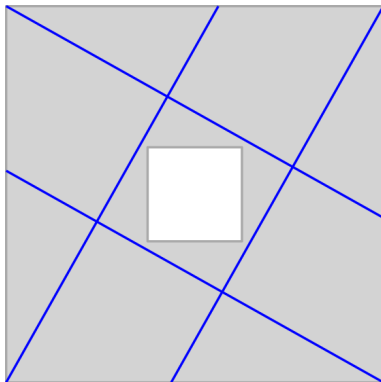


Example with a Hole



Example with a Hole

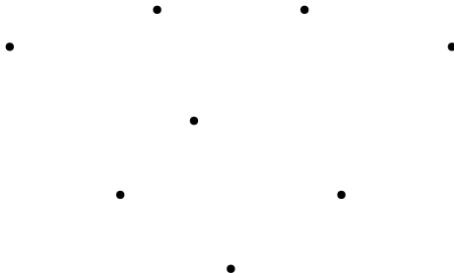


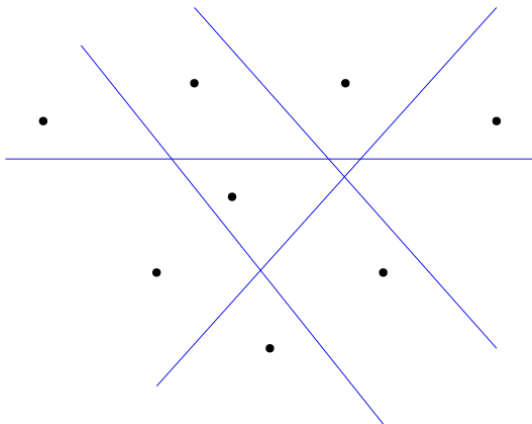


Solution to our extended version of the Skeleton Problem for a square with a hole.

- ▶ Intersects every convex vertex
- ▶ Connected
- ▶ Covers every hole

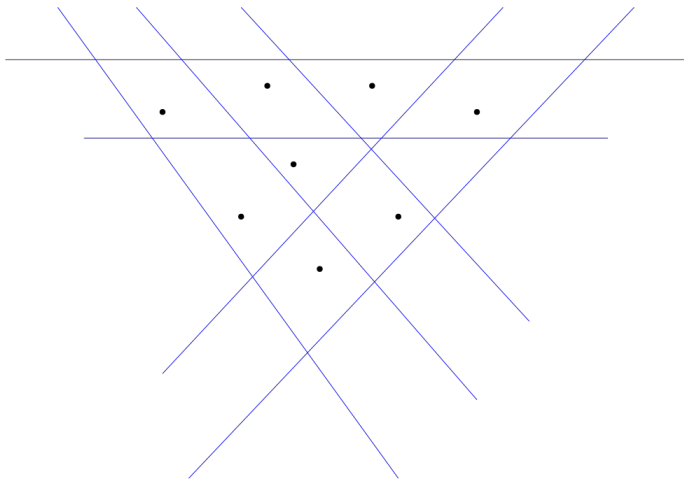
Given a set of n points in $\mathbb{R}^2$, find a minimum cardinality set of lines S such that each point is contained within the closure of some 2-D cell of S and the closure of any 2-D cell of S contains no more than one point.



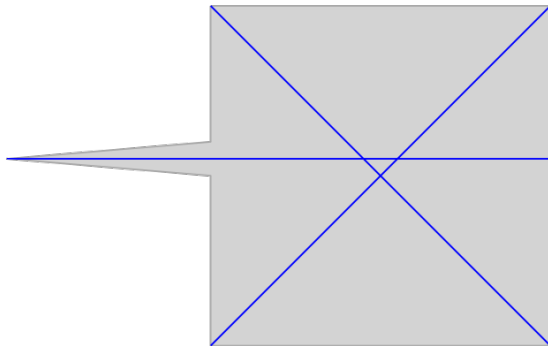


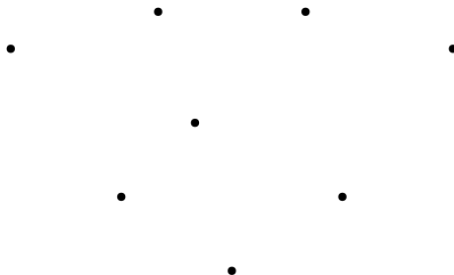
A solution to the Point Shattering problem for these points. What are the similarities between points and holes?

Point Shattering

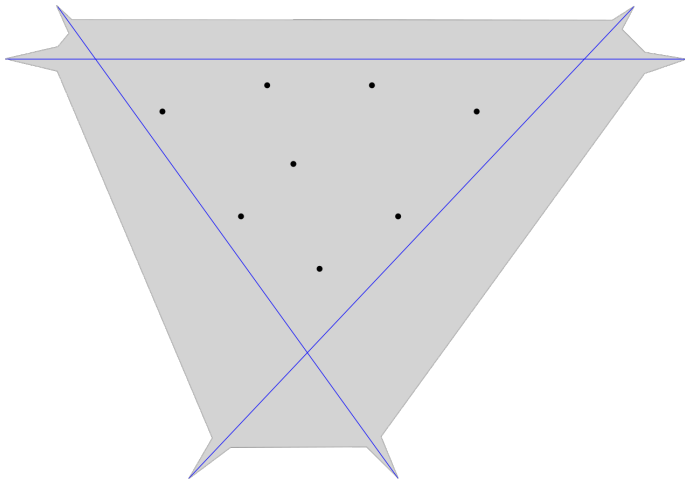


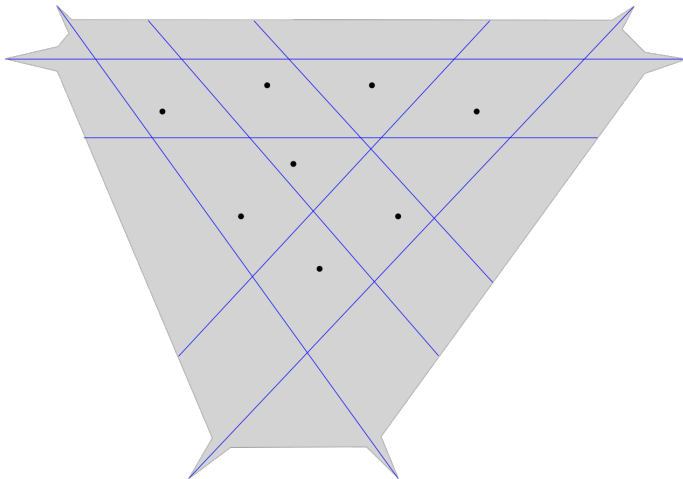
Spike Gadget

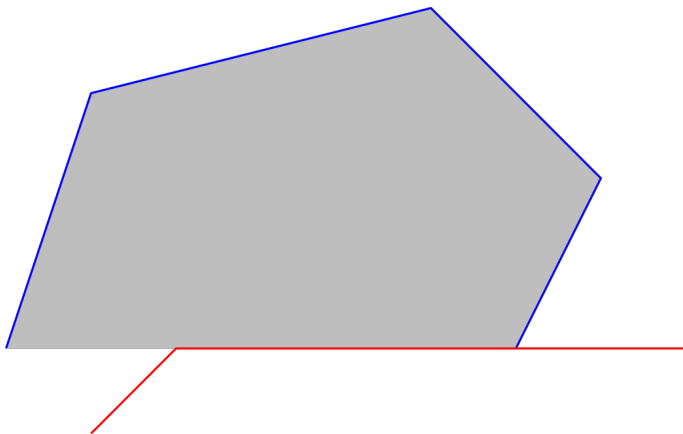




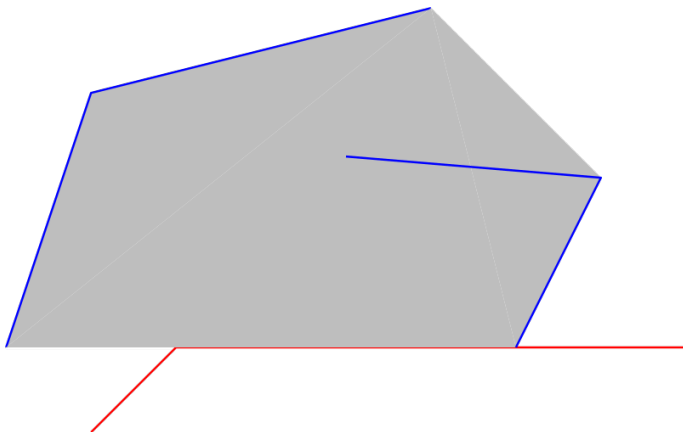
We want our polygon to force edges to surround these points in a triangle.







View of a skeleton (visibility path) from the perspective of a maximal subsegment of some boundary edge.



Why must the visibility path be connected and convex?

Claim

The union of visibility paths P for all maximal segments of some connected boundary path (either the exterior boundary or a boundary path of some hole) forms a connected set of skeleton edges.

Any Questions?